12 Apèndix II: Polinomi de Taylor

Sigui $f(x)$ una funció real prou regular.

Polinomi de Taylor de grau n entorn de $a \in \mathbb{R}$:	Reste d'ordre n:
$P_n(x) = \sum_{k=0}^{n} \frac{f^{k)}(a)}{k\,!}(x-a)^k$	$R_n(x) = \frac{f^{n+1)}(c)}{(n+1)\,!}(x-a)^{n+1} \quad c \in < a, x >$

$f(x)$		Polinomi
e^x		$P_n(x) = \sum_{k=0}^{n} \frac{x^k}{k\,!} = 1 + \frac{x}{1\,!} + \frac{x^2}{2\,!} + \cdots + \frac{x^n}{n\,!}$
$\ln(1+x)$		$P_n(x) = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}\, x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n+1}}{n}\, x^n$
$\cos(x)$	$n{=}2m$	$P_{2m}(x) = \sum_{k=0}^{m} \frac{(-1)^k}{(2k)\,!}\, x^{2k} = 1 - \frac{x^2}{2\,!} + \frac{x^4}{4\,!} - \cdots + \frac{(-1)^m}{(2m)\,!}\, x^{2m}$
$\sin(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = \sum_{k=0}^{m} \frac{(-1)^k}{(2k+1)\,!}\, x^{2k+1} = x - \frac{x^3}{3\,!} + \frac{x^5}{5\,!} - \cdots + \frac{(-1)^m}{(2m+1)\,!}\, x^{2m+1}$
$\tan(x)$	$n{=}9$	$P_9(x) = x + \frac{x^3}{3} + \frac{2\,x^5}{15} + \frac{17\,x^7}{315} + \frac{62\,x^9}{2835}$
$(1+x)^\alpha$	$\alpha\in\mathbb{R}$	$P_n(x) = \sum_{k=0}^{n} \binom{\alpha}{k} x^k = 1 + \frac{\alpha}{1!}\, x + \frac{\alpha(\alpha-1)}{2\,!}\, x^2 + \cdots + \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-(n-1))}{n\,!}\, x^n$
$\frac{1}{1+x}$	$\alpha{=}-1$	$P_n(x) = \sum_{k=0}^{n} (-1)^k\, x^k = 1 - x + x^2 - x^3 + \cdots + (-1)^n\, x^n$
$\cosh(x)$	$n{=}2m$	$P_{2m}(x) = \sum_{k=0}^{m} \frac{x^{2k}}{(2k)\,!} = 1 + \frac{x^2}{2\,!} + \frac{x^4}{4\,!} + \cdots + \frac{x^{2m}}{(2m)\,!}$
$\sinh(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = \sum_{k=0}^{m} \frac{x^{2k+1}}{(2k+1)\,!} = x + \frac{x^3}{3\,!} + \frac{x^5}{5\,!} + \cdots + \frac{x^{2m+1}}{(2m+1)\,!}$
$\tanh(x)$	$n{=}9$	$P_9(x) = x - \frac{x^3}{3} + \frac{2\,x^5}{15} - \frac{17\,x^7}{315} + \frac{62\,x^9}{2835}$
$\arctan(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = \sum_{k=0}^{m} \frac{(-1)^k}{2k+1}\, x^{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^m}{2m+1}\, x^{2m+1}$
$\text{arctanh}(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = \sum_{k=0}^{m} \frac{x^{2k+1}}{(2k+1)} = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2m+1}}{2m+1}$
$\arcsin(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = x + \sum_{k=1}^{m} \frac{(2k-1)\,!!}{2^k(2k+1)k\,!}\, x^{2k+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots + \frac{(2m-1)\,!!}{2^m(2m+1)m\,!}\, x^{2m+1}$
$\arccos(x)$		$= \frac{\pi}{2} - \arcsin(x)$
$\text{arcsinh}(x)$	$n{=}2m{+}1$	$P_{2m+1}(x) = x + \sum_{k=1}^{m} \frac{(-1)^k(2k-1)\,!!}{2^k(2k+1)k\,!}\, x^{2k+1} = x - \frac{x^3}{6} + \frac{3x^5}{40} + \cdots + \frac{(-1)^m(2m-1)\,!!}{2^m(2m+1)m\,!}\, x^{2m+1}$

Per a x propers a zero, podem agafar aquests polinomis per a aproximar les funcions.